

Enrollment No: \_\_\_\_\_

Exam Seat No: \_\_\_\_\_

# C.U.SHAH UNIVERSITY

## Winter Examination-2022

Subject Name: Engineering Mathematics-I

Subject Code: 4TE01EMT3

Branch: B.Tech (All)

Semester: 1

Date: 09/01/2023

Time: 11:00 To 02:00

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1**

**Attempt the following questions:**

**(14)**

- a) The principal argument of  $z = 1 + i$ . (01)  
(a)  $\frac{\pi}{3}$  (b)  $\frac{\pi}{4}$  (c)  $\frac{\pi}{5}$  (d)  $\frac{\pi}{6}$
- b) The conjugate of  $z = \frac{(5-3i)}{4+3i}$  is \_\_\_\_\_. (01)  
(a)  $\frac{11}{5} + \frac{27}{5}i$  (b)  $\frac{11}{5} - \frac{27}{5}i$  (c)  $\frac{27}{5} - \frac{11}{5}i$  (d)  $\frac{27}{5} + \frac{11}{5}i$
- c) The Principal value of  $\log(2 + 2i)$  is ? (01)  
(a)  $\log 2 + i\frac{\pi}{4}$  (b)  $\log 2 - i\frac{\pi}{4}$  (c)  $-\frac{1}{2}\log 2 + i\frac{\pi}{4}$  (d)  $\frac{1}{2}\log 2 + i\frac{\pi}{4}$
- d) The Eigen values of matrix  $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$  are \_\_\_\_\_. (01)  
(a) 1,3 (b) 2,3 (c) 1,4 (d) 2,4
- e) If  $y = \sin(ax + b)$  then  $y_n =$  \_\_\_\_\_. (01)  
(a)  $b^2 \sin(ax + b + n\frac{\pi}{2})$  (b)  $a^2 \cos(ax + b + n\frac{\pi}{2})$   
(c)  $a^n \sin(ax + b + n\frac{\pi}{2})$  (d)  $a^n \cos(ax + b + n\frac{\pi}{2})$
- f) If  $y = e^{3x}$  then  $y_n =$  \_\_\_\_\_. (01)  
(a)  $3^{n+1}e^{3x}$  (b)  $3^{n-1}e^{3x}$  (c)  $3^n e^{3x}$  (d)  $3^{n+1}e^{4x}$
- g) The Maclaurin's series of  $\cos x$  is \_\_\_\_\_. (01)  
(a)  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$  (b)  $1 + \frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!}$  (c)  $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$  (d)  $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$
- h) Find  $\lim_{x \rightarrow \frac{\pi}{2}} \sin x$ . (01)  
(a) 0 (b) 1 (c) -1 (d) 2
- i) Find  $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{2x^2 - 5x + 4}$ . (01)  
(a) 0 (b)  $-\frac{1}{2}$  (c)  $\frac{1}{3}$  (d)  $-\frac{1}{3}$
- j) The value of  $\lim_{x \rightarrow 1} \frac{x^3 + y^3}{x^2 + y^2}$  is \_\_\_\_\_. (01)  
 $y \rightarrow 3$



- (a)  $\frac{14}{5}$  (b)  $\frac{5}{14}$  (c) 5 (d) 14
- (k) The degree of Homogeneous equation  $f(x, y) = x^3 + x^2y + y^3$  is \_\_\_\_\_. (01)  
 (a) 2 (b) 3 (c) -2 (d) -3
- (l) The value of  $\lim_{x \rightarrow 0} \frac{3^x - 2^x}{x}$  is \_\_\_\_\_. (01)  
 (a)  $\log \frac{2}{3}$  (b)  $\log \frac{3}{2}$  (c)  $\log \frac{9}{4}$  (d)  $\log \frac{4}{9}$
- (m) For which value k vectors  $v_1 = (1, k)$ ,  $v_2 = (2k, 2)$ , are linearly dependent. (01)  
 (a)  $k = 4$  (b)  $k = 3$  (c)  $k = 1$  (d)  $k = 5$
- (n) The rank of matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 5 \end{bmatrix}$  is \_\_\_\_\_. (01)  
 (a) 3 (b) 2 (c) 1 (d) 0

**Attempt any four questions from Q-2 to Q-8**

- Q-2** **Attempt all questions** (14)
- (a) If  $y = e^{a\sin^{-1}x}$   $x \in (-1, 1)$  then prove that  $(1 - x^2)y_2 - xy_1 - a^2y = 0$ .  
 If  $y = \cos(ax + b)$   $a, b$  are constant with  $a \neq 0$  then prove that  
 (b)  $y_n = \left\{ \cos \left( ax + b + n \left( \frac{\pi}{2} \right) \right) \right\} n \in N$ .
- (c) If  $y = \frac{1}{(2x+1)(3x+1)}$ ,  $x \neq -\frac{1}{3}, -\frac{1}{2}$  then find  $y_n$ .
- Q-3** **Attempt all questions** (14)
- (a) Find a Maclaurin's series for  $f(x) = \cos x$ .  
 (b) Expand  $f(x) = \log x$  in power of  $(x - 2)$ .  
 (c) Expand  $f(x) = x^4 - 11x^3 + 43x^2 - 6x + 14$  ascending power of  $(x - 1)$ .
- Q-4** **Attempt all questions** (14)
- (a) Find  $\lim_{x \rightarrow y} \frac{x^y - y^x}{x^x - y^y}$ , where  $y$  is constant.  
 (b) Find  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$ .  
 (c) Evaluate  $\lim_{x \rightarrow 0} \frac{a^x + b^x + c^x}{3}$ .
- Q-5** **Attempt all questions** (14)
- (a) Find the roots of the equation  $z^5 = 1$ .  
 Prove that  
 (b)  $(1 + \cos\theta + i \sin\theta)^n + (1 + \cos\theta - i \sin\theta)^n = 2^{n+1} \cos^n \frac{\theta}{2} \cos \left( \frac{n\theta}{2} \right)$ .  
 (c) Express  $\frac{(1+i)(1-i)}{(4-3i)(4+3i)}$  in the form of  $a + ib$ .
- Q-6** **Attempt all questions** (14)
- (a) Verify Caley-Hamilton theorem for matrix  $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$  (06)
- (b) Find the Eigen Values of the matrix  $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 4 & 5 \\ 1 & 4 & 6 \end{bmatrix}$  (05)



- (c) Find the rank of matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$  (03)

**Q-7** **Attempt all questions** (14)

- (a) Find the value of  $\lim_{x \rightarrow 0, y \rightarrow 0} \frac{x^3 - y^3}{x^2 + y^2}$ ,  $x \neq 0, y \neq 0$  (05)

- (b) If  $u = e^{xyz}$  then prove that  $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$ . (05)

- (c) If  $u = \tan^{-1} \frac{y}{x}$  then find  $\frac{\partial^2 u}{\partial x \partial y}$ . (04)

**Q-8** **Attempt all questions** (14)

- (a) Express  $\cos 8\theta$  in terms of  $\cos \theta$  and  $\sin \theta$ . (06)

- (b) Solve system of linear equations by using Gauss-elimination method (05)

$$3x + y - z = 3, x - 2y + 9z = 8, 2x - 8y + z = -5.$$

- (c) Evaluate  $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$ . (03)

